ON THE MOTION OF A BODY UNDER THE ACTION OF AN EXTERNAL ENERGY SOURCE

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We present a theoretical analysis of the problem of the acceleration of a body of revolution by means of a reactive gas jet with energy supplied from an external source. Heating of the gases flowing out through a nozzle in the tail portion of the body can be accomplished with a laser beam. The reactive thrust has been calculated in a one-dimensional approximation.

1. We consider the problem concerning the motion in a gas or in a vacuum of a body M of mass m_0 for the case in which the motion of M is due to an external energy source or to an internal energy source with a mass negligible in comparison with m_0. For definiteness we assume that the body has a cylindrical shape and is bounded at the one end by a flat circular face or by a cone with a small spherically blunt end (the head portion), and at the other end (the tail portion) it is assumed that the body has an indentation in the shape of a surface of revolution.

Choosing the head portion of the body to have a flat end shape furnishes an example of a poorly streamlined body, while the choice of a blunted cone helps to diminish the aerodynamic resistance which the body encounters in the air at high speeds [1]. A reactive jet is produced in the tail portion of the body and the shape of the indentation there must then accommodate the larger thrust obtained.

At an initial instant t = 0 let energy $E_0(t)$ by supplied to the tail indentation (nozzle). This energy can be supplied, for example, by a laser beam, the focussing of an electron beam, radio band electromagnetic waves, and by other means. The resultant heating of the gases entering the nozzle or the vaporization of the nozzle walls gives rise to a nonstationary flow. The heated gas or the vaporization products from the walls of the tail chamber expands and flows out into the surrounding space. The resulting reaction force then sets the body into motion.

2. We consider approximate methods for calculating the acceleration, thrust, and specific impulse which arise for two basic models of the processes occurring in the tail nozzle.

Model 1. M moves under the action of reactive forces arising during the heating and expansion of the gases entering the tail nozzle from the forward and middle portions of the body.

Model 2. M moves as the result of the reactive forces which arise as the vaporization products from the nozzle walls flow out.

According to Newton's second law

$$m\frac{d\mathbf{U}}{dt} = \mathbf{F}_e, \tag{1}$$

where $\mathbf{U}$ is the velocity of the body M, m is its mass at time t, and $\mathbf{F}_e$ represents the resultant external forces acting on the body M.

Taking the motion of the body vertically upwards with respect to the earth's surface, we assume that $\mathbf{F}_e$ is directed vertically, where

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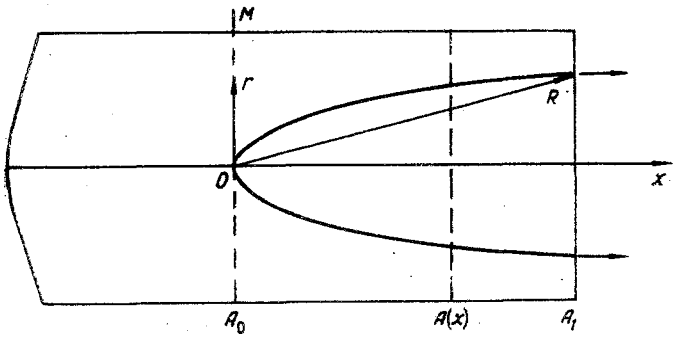

Fig. 1. Geometric parameters of the tail section of the body.

$$F_{\rho} = -mg - F_D + F_T; \tag{2}$$

here F_T is the thrust, i.e., the reactive force acting on M; F_D is the air resistance; and g is the gravitational acceleration. From gas dynamics it is known (see, for example, [2, 3]) that if v_1 is the gas velocity, averaged over the exterior end section of the nozzle, and Q is the mass outflow rate of the gases, then the nozzle thrust F_T is given by the simple expression

$$F_T = Qv_1 + \Delta PA_1, \tag{3}$$

where ΔP is the excess pressure at the exterior section of the nozzle; A_1 is the cross-sectional area of the exterior section.

Using a quasi-one-dimensional approximation for flow in the nozzle, we can write [2, 3]

$$Q = \int_A \rho v_n dA \approx \rho(x, t)\, v(x, t)\, A(x),$$

where A(x) is the cross-sectional area of the nozzle at the distance x from its narrowest section; ρ is the average gas density at the section A; v_n is the normal component (on A) of the gas velocity (see Fig. 1). The specific impulse I_{sp} can be defined as follows:

$$I_{sp} = \frac{F_T}{Qg}. \tag{4}$$

In what follows we neglect, for simplicity, the influence of gravity and the air resistance force F_D. Then for Q = const, v_1 = const, $\Delta P = 0$, we obtain from the relations (1)-(3)

$$\frac{dU}{dt} = \frac{Qv_1}{m}. \tag{5}$$

If the nozzle outflow rate takes place at the expense of a mass loss in the body M, we can write $m = m_0 - Qt$, and, in accordance with relation (5), we have the known expression (see, for example, [4])

$$U = v_1 \ln \frac{m_0}{m} \tag{6}$$

(we have here taken into account the fact that U = 0 for t = 0).

When variation in the mass of the body M can be neglected, we have, from equation (5), the result

$$U = \frac{Qv_1}{m_0} t. \tag{7}$$

3. For flow in the nozzle according to Model 1 we have the following two cases:

a) A continuous supply of gas goes on at the section A_0 (for example, with the aid of an air intake) and a pressure P_0 is maintained at the expense of external heating.

b) At the section A_0, the linear dimensions of which are negligibly small in comparison with the transverse size of the body M, a quantity of energy E_0 is injected instantaneously at the time t = 0 and the nozzle is filled with a quiescent gas or a gas in stationary motion.

We consider the problem of determining the nozzle parameters in the case a) when the pressure P_0 is given, the flow is stationary, and P_0 is close to the value of the retardation pressure. For a flow tube we have, in accordance with Bernoulli's equation,

$$v_1 = \sqrt{\frac{2\gamma}{\gamma-1}\left(\frac{P_0}{\rho_0}\right)\left[1-\left(\frac{P_1}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}, \tag{8}$$

where ρ_0 is the density at the section A_0; γ is the effective exponent of adiabaticity; and P_1 is the pressure at the outer section of the nozzle. For the density ρ_1, assuming adiabaticity, we have

$$\rho_1 = \rho_0\left(\frac{P_1}{P_0}\right)^{\frac{1}{\gamma}}. \tag{9}$$

Relations for averaged flows were given in [2] under more general assumptions.

We can assume the pressure P_1 at the outer section of the nozzle to be given. If we assign P_0 and ρ_0 (or the temperature at the section A_0, where the speed is small), then we can determine from the equations (8) and (9) the outflow speed v_1 and the density ρ_1. For a stationary flow the energy flow of gas in the nozzle must be equal to the energy supplied by the laser or by other means. From the integral energy conservation law we have the relationship

$$I_{01} - I_{00} = \int_{x_0}^{x_1} a^{(e)} A\,dx + \int_{x_0}^{x_1} q^{(e)} A\,dx, \tag{10}$$

where

$$I_0 = \int_A \left(\varepsilon + \frac{v^2}{2} + \frac{P}{\rho}\right)\rho v_n\,dA;$$

$a^{(e)}$ and $q^{(e)}$ are, respectively, the specific work of the external forces and the specific external heat input; I_{01} and I_{00} are the total heat content flows at the sections $A(x_1)$ and A_0, respectively. If, for simplicity, we neglect I_{00} and $a^{(e)}$, and if we take $\int_{x_0}^{x} q^{(e)} A\,dx$ proportional to the cross-sectional area of the beam and the intensity of theincident radiation, then, as an estimate of the energy supplied, we can write

$$Q\left[\frac{\gamma P_1}{(\gamma-1)\rho_1} + \frac{v_1^2}{2}\right] = S_b W.$$

Here S_b is the beam cross-sectional area; W is the specific flow of energy absorbed by the gas. From this we obtain

$$W = \rho_1 v_1\left(\frac{\gamma P_1}{(\gamma-1)\rho_1} + \frac{v_1^2}{2}\right)\frac{S_1}{S_b}.$$

If we take $\gamma = 1.25$, $S_1/S_b = 10$, $\rho_1 \approx 0.1\cdot10^{-3}$ g/cm^3, $P_1 = 1$ atm, $v_1 = 3\cdot10^5$ cm/sec (this corresponds to $P_0 \approx 20$ atm, $\rho_0 = 10^{-3}$ g/cm^3), then from Eq. (10) we obtain $W \approx 5\cdot10^6$ W/cm^2. Such energy flow densities are attainable with present-day lasers [8]. Since, with losses taken into account, these power densities must be increased by roughly an order of magnitude, there arises the problem of avoiding air breakdown along the laser beam path and also the preliminary ionization of the gas in the tail chamber to facilitate breakdown of the gas in the vicinity of the section A_0 and to increase the coefficient of absorption of the energy supplied. These problems will not be discussed here. If we take $A_1 = 3\cdot100$ cm^2, then for the parameter values indicated above we obtain, from the relations (3) and (4) with $\Delta P = 0$,

$$F_r \approx 10^{10}\ \text{dyn},\quad I_{sp} \approx 3\cdot10^2\ \text{sec}.$$

From relation (1) we find that the escape time to cosmic velocity for a body of mass 10^5 g is around 8 sec.

Case b). When the process in the nozzle is nonstationary, we must solve the following problem. At the section A_0, at the initial instant, an input of mass Q(t) and energy E(t) commences. We shall consider a conical nozzle. The initial density in the nozzle is assumed to be either constant or distributed according to the law $\rho_e = n/R^2$ (see [6, 7]). As a result of the energy and mass input to the gas, a nonstationary flow commences. To determine the parameters of this flow we need to solve the nonstationary gasdynamic equations for a one-dimensional spherically symmetric flow.

We consider its solution only in the limiting case of negligibly small mass and an instantaneous deposition of energy E_0 at the section A_0. Let $\rho_e = n/R^{\omega}$, and let the cone angle be small so that $x \approx R$, where R is the radius vector from the point 0 (see Fig. 1). If we neglect counterpressure, the problem obtained is that of a strong explosion, the solution for which is known [2].

The density and velocity distributions behind the shock wave that is formed are given by the approximate expressions (which are exact for $\omega = 2$, $\gamma = 5/3$)

$$v = \frac{2\delta}{\gamma+1}\frac{R}{t}, \qquad \rho = \frac{\gamma+1}{\gamma-1}\rho_e\left(\frac{R}{r_*}\right)^s, \tag{11}$$

$\delta = 2/5 - \omega$; $s = 3/(\gamma-1)$ for $\omega = 0$; $s = 1$ for $\omega = 2$, $\gamma = 5/3$; $r_* = (E_0/n)^{\delta/2}t^{\delta}$; and E is a quantity proportional to the energy supplied. The outflow rate Q through the section $A_1 = A(x_1)$ of radius r_1 can be determined approximately as

$$Q = \pi r_1^2 \rho(x_1, t)\, v(x_1, t), \tag{12}$$

$$t > t^* = x_1^{\frac{1}{\delta}}\left(\frac{n}{E}\right)^{1/2}.$$

Here the values of $\rho(x_1, t)$, $v(x_1, t)$ are taken from the relations (11) for $R = x_1$.

Taking into account the relations (12), (11), and (3), we can determine the nonstationary thrust acting on the body M. We present an approximate formula for the specific impulse:

$$I_{sp} = \frac{2\delta}{g(\gamma+1)}\frac{x_1}{t} - \frac{\Delta P}{g\rho_1 v_1}, \qquad t > t^*.$$

For $E = 100$ J, with $x_1 = 10^2$ cm, we have $t = 0.2$ sec, $\gamma = 1.2$ ($t^* = 0.1$ sec), $\rho = n = 10^{-3}$ g/cm^3, $I_{sp} \approx 0.3$ sec ($\omega = 0$); also $t = 0.1$ sec ($t^* = 0.03$ sec), $\gamma = 5/3$, $n = 1$ g/cm^3, $I_{sp} \approx 1.5$ sec ($\omega = 2$).

These numbers show that to obtain typical impulses $I_{sp} \gg 1$, we need to take $x_1 < 10^2$ cm or to supply energy greater than 100 J (in the numerical data above the energy $E_0 = E/\alpha$, where $\alpha < 1$ in the case of small cone angles, but this does not improve substantially the estimates for I_{sp}).

The estimates presented above were obtained in a quasi-stationary approximation. An exact account of nonstationarity in a formula of the type (3) also changes the quantity I_{sp} somewhat.

We note also that the time-averaged impulse will be somewhat higher than the instantaneous impulse.

4. For the Model 2, let us now suppose that the gases flowing out of the tail nozzle are formed as a result of vaporization of the forward wall of the nozzle during heating by a laser beam focussed on the area A_0. For the velocity v_0 of the vapors flowing out we use the expression [8]

$$v_0 = \frac{kW}{\rho_0[L + C(T_b - T_0)]} \qquad (1 < k < 15),$$

where W is the absorbed power of the electromagnetic source, L is the specific heat of vaporization of the material with density ρ_0; C is the heat capacity of the material; T_b is the vaporization temperature; T_0 is the ambient temperature. If we take T_b and ρ_0/k as retardation parameters, then, using the relations (8) and (9), we can estimate the thrust and impulse from the scheme indicated above. However, since a substantial mass loss of the body M can occur here, we must start from equation (6) in determining the velocity of the body M.

Let Q be the material mass being vaporized per unit time. Taking this mass as the quantity being discharged from the nozzle, and knowing v_1, we can determine the thrust F_T.

To illustrate, we consider the case in which the nozzle tail surface is covered with lead. Then, using known data [8], we find for the case $W = 2 \cdot 10^7$ W/cm^2, $A_0 = 10^{-3}$ cm^2, $v_0 = 3 \cdot 10^4$ cm/sec, $Q = 30$ g/sec. Taking the outflow velocity v_1 equal to $2 \cdot 10^5$ cm/sec, we find a thrust $F_T = 6 \cdot 10^6$ dyn ≈ 6 kg. From Eq. (6) we find that the time taken to attain the velocity U is

$$t = \frac{m_0}{Q}\left[1 - e^{-\frac{U}{v_1}}\right].$$

From this, with $m_0 = 100$ g, we find that the time taken to attain cosmic velocity is around 3 sec, during which time 98% of the mass of the body will have been spent.

The calculations given here show that, in principle, a body of small mass can be accelerated to cosmic velocities with the aid of external energy sources. The principles considered for such accelerations can be used for modelling the motion of meteorites in the earth's atmosphere and in the atmospheres of other planets.

We note, in conclusion, that our results correspond, in part, to the schemes for the acceleration of bodies that were mentioned in [5] but with no supporting calculations.

NOTATION

U	is the body velocity;
F_T	is the reactive force;
v_1	is the gas outflow velocity;
I_{sp}	is the specific mechanical impulse;
Q	is the flow-rate;
W	is the absorbed power of laser radiation;
x	is the coordinate along the nozzle axis;
$A(x)$	is the cross-sectional area.

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